

Flat direction condensate instabilities in the MSSM

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Abstract

Coherently oscillating scalar condensates formed along flat directions of the MSSM scalar potential are unstable with respect to spatial perturbations if the potential is flatter than ϕ^2 , resulting in the formation of non-topological solitons such as Q-balls. Using the renormalization group we calculate the corrections to the ϕ^2 potential for a range of flat directions and show that unstable condensates are a generic feature of the MSSM. Exceptions arise for an experimentally testable range of stop and gluino masses when there are large admixtures of stops in the flat direction scalar.

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1 Introduction

The scalar potential of the Minimal Supersymmetric Standard Model (MSSM) [1] is a complicated function of 45 complex squark and slepton fields and 4 complex Higgs fields. A natural feature of this potential is the existence of flat directions, corresponding to linear combinations of these fields such that there are no renormalizable contributions to the scalar potential along the flat direction beyond the soft SUSY breaking terms [2]. These flat directions may play a fundamental role in the cosmology of the MSSM, in particular as a natural source of the baryon asymmetry of the Universe via the Affleck-Dine mechanism [3], in which a baryon asymmetry is induced in a coherently oscillating condensate of squarks and sleptons. It has recently been realized that the cosmology of flat directions of the MSSM and Affleck-Dine baryogenesis may be more complicated than previously thought [4, 5, 6, 7, 8]. If the flat direction scalar potential increases less rapidly than ϕ^2 , where ϕ is the scalar field along the flat direction ("Affleck-Dine (AD) scalar"), then once coherent oscillations of the field begin, the condensate will have a negative pressure making it unstable with respect to spatial perturbations [5, 7]. These spatial perturbations, which generally arise during inflation as a result of quantum fluctuations of the AD scalar [7, 8, 9], will grow and go non-linear, fragmenting the condensate into lumps which eventually evolve into Q-balls of baryon number ("B-balls") [5, 7, 9]. The formation of Q-balls has recently been verified by lattice simulations [10].

The cosmological evolution of the resulting Q-balls depends on the form of SUSY breaking. Q-balls from the AD condensate were first discussed in the context of gauge-mediated SUSY breaking [5, 6], in which the flatness of the potential occurs above the mass of the messenger fields. It was later realized that Q-balls can also form in the more conventional gravity-mediated SUSY breaking models [7, 8]. In this case the potential is likely to be flatter than ϕ^2 as a result of radiative corrections. The resulting Q-balls are not stable but can be very long-lived, decaying after the electroweak phase transition has occurred [8]. As a result Q-balls can protect the baryon asymmetry from the effect of lepton number violating interactions combined with sphaleron processes

and, if they decay at a low enough temperature, can also act as a common source of baryons and dark matter neutralinos, so explaining the similarity of the number densities of baryons and dark matter particles when the dark matter particles have masses of the order of m_W [8, 11]. They can also enhance the isocurvature density perturbations expected from AD baryogenesis in particular inflation models [12, 13].

The existence and cosmology of Q-balls in the MSSM with gravity-mediated SUSY breaking is dependent upon the details of the radiative correction to the flat direction scalar potential. The correction must be negative in sign relative to the SUSY breaking mass squared term in order for Q-balls to form, whilst the binding energy of the charges within the Q-ball and so the total charge and lifetime of the Q-ball depends upon the magnitude of the correction [8], as does the fraction of the total baryon number initially trapped within the Q-balls [9]. In this letter we consider in detail the radiatively corrected scalar potential for a range of flat directions using the renormalization group.

We will show that condensate collapse and Q-ball formation are almost a general feature of all the MSSM flat directions, with exceptions only for the cases of $d=4$ $H_u L$ -direction and directions with large admixtures of stop. The existence of instabilities along the latter directions depend on the mass of the stop in a testable way, as will be discussed in the following.

F- and D-flat directions of the MSSM have been classified and listed in [2]. For gravity-mediated SUSY breaking the scalar potential along a flat direction has the form [7, 8]

$$U(\Phi) \approx m^2 \left(1 + K \log \left(\frac{|\Phi|^2}{M^2} \right) \right) |\Phi|^2 + \frac{\lambda^2 |\Phi|^{2(d-1)}}{M_*^{2(d-3)}} + \left(\frac{A_\lambda \lambda \Phi^d}{d M_*^{d-3}} + h.c. \right), \quad (1)$$

where m is the conventional gravity-mediated soft SUSY breaking scalar mass term ($m \approx 100$ GeV), K is a parameter which depends on the flat direction, and d is the dimension of the non-renormalizable term in the superpotential which first lifts the degeneracy of the flat direction; it is of the form $f = \lambda M_*^{4-d} \Phi_1 \cdots \Phi_d$. We assume that the natural scale of the non-renormalizable terms is M_* , where $M_* = M_{Pl}/\sqrt{8\pi}$ is the supergravity mass scale.

The equation of state for a field oscillating in a potential ϕ^γ reads as [14] $p =$

$(\frac{2\gamma}{\gamma+2} - 1)\rho$ so that Eq. (1) gives rise to the equation of state $p = \frac{K}{2}\rho$. With $K < 0$ the pressure is negative and hence the AD condensate is unstable. The condensate fragments and eventually the lumps will evolve dynamically into the state of lowest energy, the Q-ball. If $K > 0$ the condensate is stable and no Q-balls will form. Although a negative K should be a generic feature of the MSSM, not all flat directions will have negative K for all the values of the MSSM parameters. Moreover, as the actual value of K dictates the dynamical evolution of the AD condensate and its fragmentation, it is of great interest to find out the precise value of K for a given flat direction and for a range of the MSSM parameter values.

K can be computed from the RG equations, which to one loop have the form

$$\frac{\partial m_i^2}{\partial t} = \sum_g a_{ig} m_g^2 + \sum_a h_a^2 (\sum_j b_{ij} m_j^2 + A^2) , \quad (2)$$

where a_{ig} and b_{ij} are constants, m_g is the gaugino mass, A is the A-term, h_a the Yukawa coupling, and $t = \ln M_X/\mu$. The full RG equations are listed in [1] and we do not reproduce them here. We assume unification at $t = 0$ and neglect all other Yukawa couplings except the top Yukawa $h_t(M_W) = 1$ (for definiteness, we choose $\tan\beta = 1$). We shall use quantities scaled by m and denote m_g/m at $t = 0$ by ξ . The potential along the flat direction is then characterized by the amount of stop mixture (where appropriate), the values of ξ and A , and in the special case of the d=4 $H_u L$ -direction, on the $H_u H_d$ -mixing mass parameter μ_H .

The mass of the AD scalar ϕ is the sum of the masses of the squark and slepton fields ϕ_i constituting the flat direction, $m_S^2 = \sum_a p_i^2 m_i^2$, where p_i is the projection of ϕ along ϕ_i , and $\sum p_i^2 = 1$. The parameter K is then given simply by

$$K = \left. \frac{\partial m_S^2}{\partial t} \right|_{t=\log\mu} . \quad (3)$$

To compute K , we have to choose the scale μ . The appropriate scale is given by the value of the AD field when it first begins to oscillate at $H \approx m$. Let the value of the mean field be ϕ_0 ; the value of K at this scale then determines whether the condensate is unstable or not. We may compute ϕ_0 from Eq. (1) by ignoring the radiative correction (i.e. setting effectively $K = 0$) and minimizing the U(1) symmetric part of U (or

neglecting the A-term in Eq. (1)). One then finds

$$\mu = |\phi_0| = \left[\frac{m^2 M_*^{2(d-3)}}{(d-1)\lambda^2} \right]^{\frac{1}{2(d-2)}}, \quad (4)$$

where in what follows we assume for simplicity that $\lambda = 1$.

Table 1. The flat directions considered

direction	dimension	mass ²
$H_u L$	4	$\frac{1}{2}(m_H^2 + \mu_H^2 + m_{\tilde{L}_i}^2)$
$uude$	4	$\frac{1}{4}(m_{\tilde{u}_i}^2 + m_{\tilde{u}_j}^2 + m_{\tilde{d}_k}^2 + m_{\tilde{e}_l}^2); (i \neq j)$
$QQQL$	4	$\frac{1}{4}(m_{\tilde{Q}_i}^2 + m_{\tilde{Q}_j}^2 + m_{\tilde{Q}_k}^2 + m_{\tilde{L}_l}^2); (i \neq j \text{ or } k)$
$(udd)^2$	6	$\frac{1}{3}(m_{\tilde{u}_i}^2 + m_{\tilde{d}_j}^2 + m_{\tilde{d}_k}^2); (j \neq k)$
$(QLd)^2$	6	$\frac{1}{3}(m_{\tilde{u}_i}^2 + m_{\tilde{d}_j}^2 + m_{\tilde{Q}_k}^2)$

The flat directions we have studied are listed in Table 1. There is one purely leptonic direction, $H_u L$, a purely baryonic one involving only the squarks, and directions which have both squark and lepton fields.

In Fig. 1 we show the contours of K for the d=4 $uude$ and $QQQL$ directions in the (A, ξ) -plane; Fig. 2 is for the d=6 $(udd)^2$ and $(QLd)^2$ directions. These should be representative of all the other directions, too, except for $H_u L$. For $\xi \sim \mathcal{O}(1)$, typical value for K is found to be about -0.05 . For all the squark directions with no stop, as long as h_b and h_u can be neglected, K is always negative, and the contours of equal K do not depend on A . This is evident from the RGEs Eq. (2). However, as far as flat directions are concerned, all squarks are equal, and having no stop mixture would appear rather unnatural. Therefore we have considered the effect of stop mixing in the squark directions, which results in A -dependence of the K -contours, as is depicted in Figs. 1 and 2. In the presence of stop mixing $K < 0$ is no longer automatic even in the purely squark directions. The more there is stop, the larger value of ξ is required for $K < 0$. However, even for pure stop directions, positive K is typically obtained only for relatively light gaugino masses with $\xi \lesssim 0.5$. In d=4 directions the effect of stop mixture is less pronounced than in the d=6 directions, as can be seen from Figs. 1 and 2.

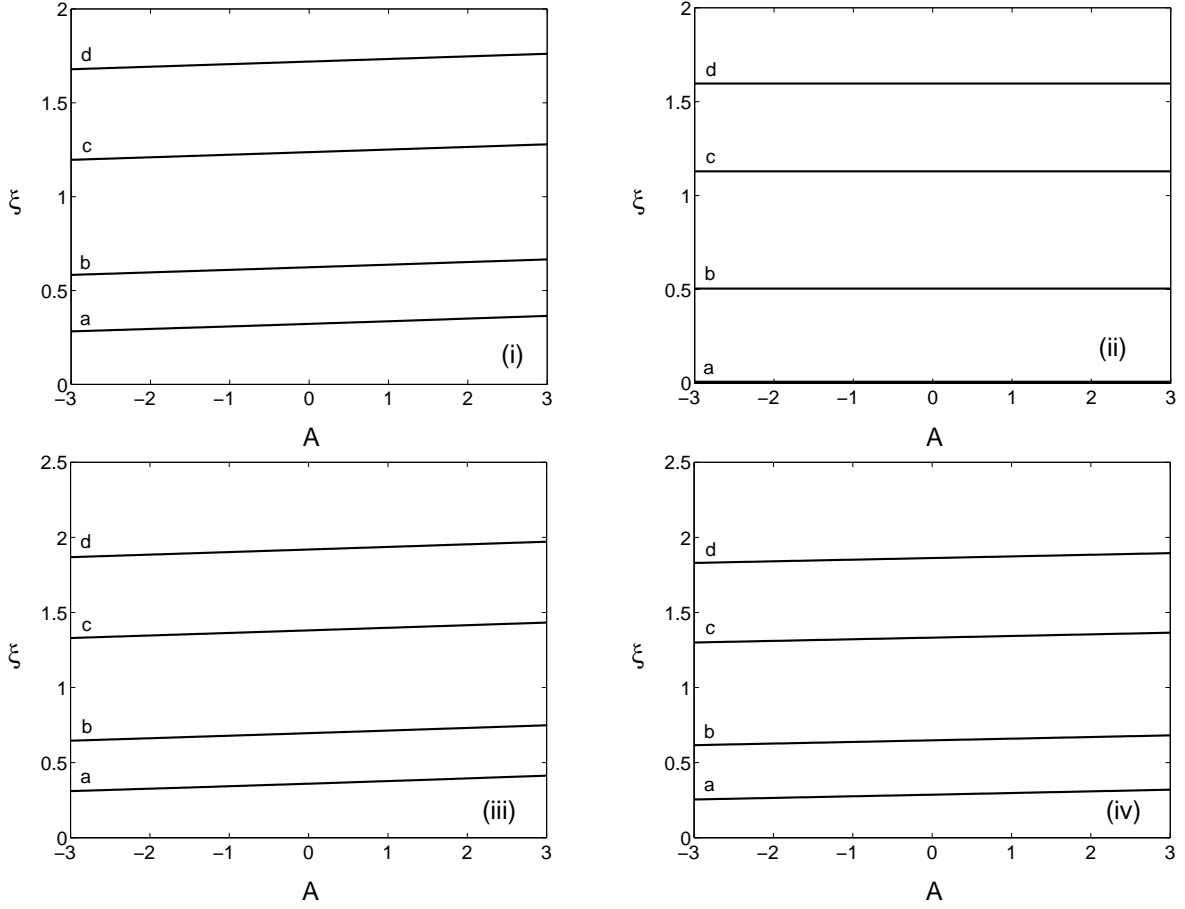


Figure 1: Contours of K for two $d=4$ flat directions: (a) $K = 0$; (b) $K = -0.01$; (c) $K = -0.05$; (d) $K = -0.1$. The directions are (i) Q_3Q_3QL ; (ii) $QQQL$, no stop; (iii) u_3ude ; (iv) $uude$ with equal weight for all u -squarks.

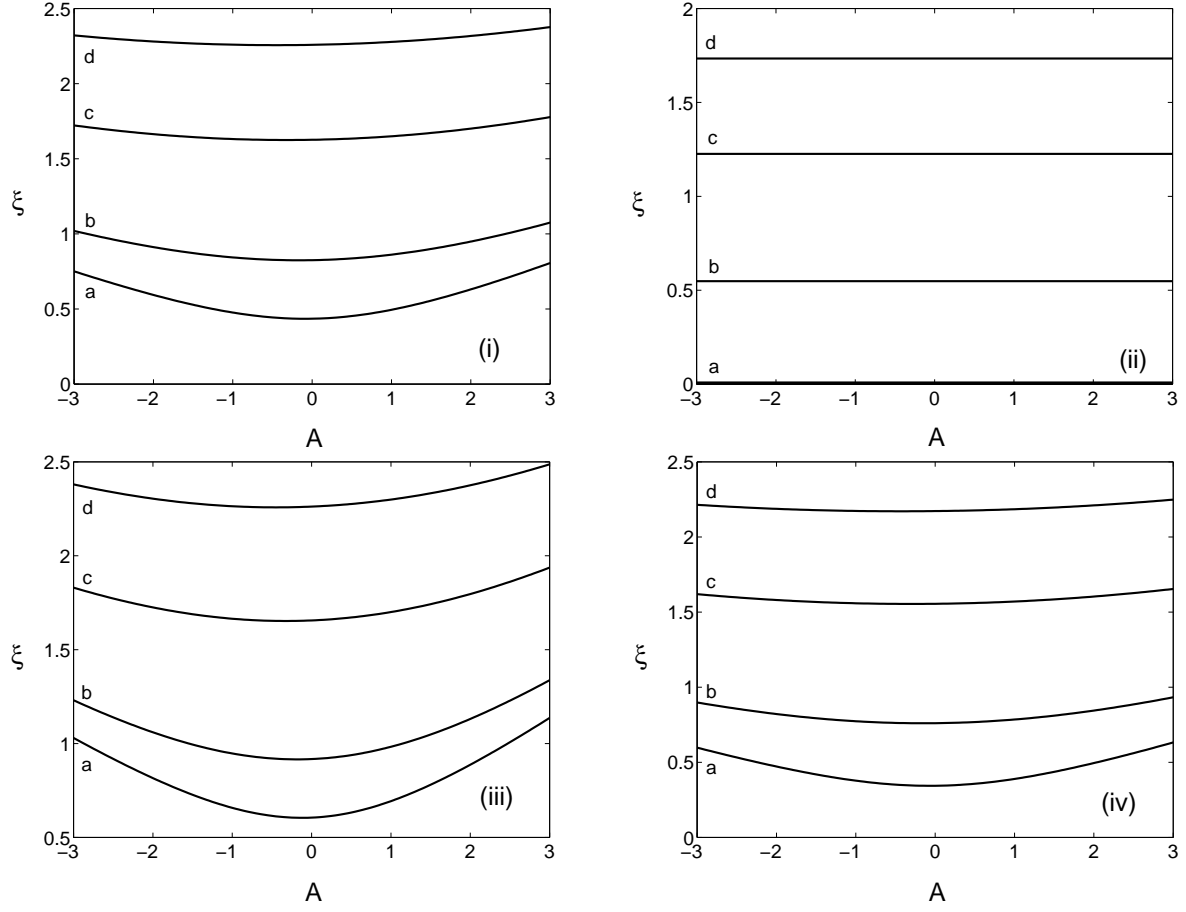


Figure 2: Contours of K for two $d=6$ flat directions: (a) $K = 0$; (b) $K = -0.01$; (c) $K = -0.05$; (d) $K = -0.1$. The directions are (i) $Q_3 L d$; (ii) $Q L d$, no stop; (iii) $u_3 d d$; (iv) $u d d$ with equal weight for all u -squarks.

In contrast to the squark directions, K was found to be always positive in the $H_u L$ -direction. This is due to the fact $H_u L$ does not involve strong interactions which in other directions are mainly responsible for the decrease of the running scalar masses. The value of μ_H was chosen in such a way that for each value of A and ξ , electroweak symmetry breaking is obtained at the scale M_W .

Except for the $H_u L$ -direction, the instability of the AD condensate is thus seen to depend on the amount of stop mixture in the flat direction. Since it would be natural to expect roughly equal mixtures of the different squark flavours in the flat direction scalar, this means that in principle instabilities, and hence Q-balls in the case of gravity mediated susy breaking, could be ruled out experimentally at LHC by measuring the mass of the gluino and the stop. To illustrate this, in Fig. 3 we show the domains of positive and negative K in the region of $(m_{\tilde{t}}, m_{\tilde{g}})$ -plane corresponding to the range $-3 < A < 3$ and $0 < \xi < 2.5$. The Figure is for the $(udd)^2$ direction with equal weight for all u -squarks. For most part $K > 0$ and $K < 0$ regions can be separated, although there is a small area below the upper $K = 0$ contour where both values can be found. For a fixed m , the $K = 0$ contour has endpoints which correspond to $A = \pm 3$; if one were to allow for a wider range in A , this would spread the region between the dashed lines towards the lower right-handed corner. Changing the value of m would redefine the physical mass scale by a factor $m/100$ GeV. Thus measuring $m_{\tilde{t}}$ and $m_{\tilde{g}}$ would not alone be sufficient to determine the existence of instabilities. In addition, one needs the values of m and $\tan\beta$ which naturally will be measured if supersymmetry will be found. Very roughly, instability is found when $m_{\tilde{g}} \gtrsim m_{\tilde{t}}$, although the exact condition should be checked case by case.

In conclusion, we have shown that the MSSM scalar condensates in all but the $H_u L$ flat direction are unstable for a large part of the parameter space. Therefore the existence of Q-balls is a generic feature in all the models that incorporate both the MSSM and inflation. Moreover, as the existence of instabilities can in principle be ruled out by measuring the mass of the stop and the gluino, one may soon be able to subject the Affleck-Dine scenario to a direct test.

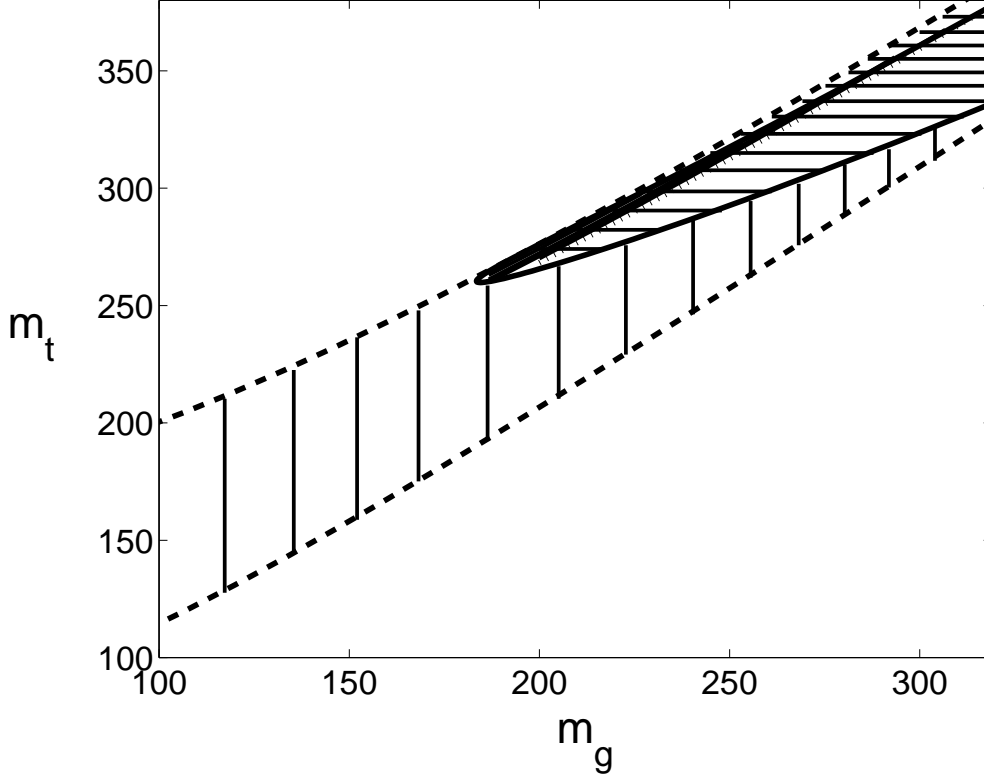


Figure 3: The regions of positive and negative K in the $(\tilde{m}_t, m_{\tilde{g}})$ -plane in the interval $-3 < A < 3$ and $0 < \xi < 2.5$ (between the dashed lines) for the udd -direction with equal weight for all u -squarks. The solid line is the $K = 0$ contour, horizontally hatched area is where $K < 0$, vertically hatched area is where $K > 0$, and there is a small region below the upper $K = 0$ -line where both $K > 0$ and $K < 0$ can be found. The units are $(m/100 \text{ GeV}) \text{ GeV}$.

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